Relatio s getween TMDs and Chiral Odd GPDs from $\pi^{\circ}$ and $\eta$ proutuction
$2^{\text {nd }}$ PQCR Workshop Jeffrison Lab

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## Outline

1) Relations between TMDs and GPDs: unraveling new multiparton correlations
2) How reliably can GPDs be measured? Towards a global fit: models, parameters, theoretical errors, resolution (GGL, PRD 2011)
Can we understand flavor decomposition of Dirac and Pauli form factors?
3) Exclusive $\pi^{0}$ electroproduction $\rightarrow$ chiral odd sector (Ahmad et al. PRD 2009, Goldstein et al., hep-ph/1201.6088)

# Extraction of GPDs from experimental data 

$\rightarrow$ Define
"what type of information"
$\rightarrow$ Define
"the way to access it"


$$
f_{\Lambda_{\gamma}, \Lambda ; \Lambda_{\gamma}^{\prime}, \Lambda^{\prime}}=\epsilon_{\mu}^{\Lambda_{\gamma}} T_{\Lambda \Lambda^{\prime}}^{\mu \nu} \epsilon_{\nu}^{* \Lambda_{\gamma}^{\prime}}
$$

## Quark-Proton Helicity Amplitudes

$$
\begin{aligned}
& f_{++}^{S}=f_{++,++}+f_{-+,-+} \\
&=g_{++}^{S} \otimes\left(A_{++,++} A_{-+,-+}\right) \\
& f_{++}^{A}=f_{++,++} f_{-+,-+} \\
&=g_{++}^{A} \otimes\left(A_{++,++}-A_{-+,-+}\right) \\
& f_{+-}^{S}=f_{++,+-+f_{-+,--}} \\
&=g_{++}^{S} \otimes\left(A_{-+,++}+A_{++,-+}\right) \\
& f_{+-}^{A}=f_{++,+-}^{\prime}-f_{-+,--}^{\prime}=\mathrm{q}+\Delta \\
&=g_{++}^{A} \otimes\left(A_{-+,++}-A_{++,-+}\right) \\
& \\
& g_{++}^{++} \pm g_{++}^{--}=\sqrt{X(X-\zeta)}\left(\frac{1}{X-\zeta+i \epsilon} \pm \frac{1}{X-i \epsilon}\right)
\end{aligned}
$$

## transversity state

GPDs and TMDs with

1. same helicity/transversity structure
2. Same parton correlations in a suitably defined forward limit


## $\mathrm{H}_{T} \leftrightarrow h_{1}$



GPDs and TMDs with

1. same helicity/transversity structure
2. Same parton correlations in a suitably defined forward limit
$\tilde{\mathrm{H}}_{T} \leftrightarrow h_{1 T}^{\perp}$

$\tilde{\mathrm{E}} \leftrightarrow g_{1 T}^{\perp}$

$\tilde{\mathrm{E}}_{T} \leftrightarrow h_{1 L}^{\perp}$


SSA Sector: GPDs and TMDs with same helicity/transversity structure

$$
\begin{aligned}
& 2 \tilde{\mathrm{H}}_{T}+\mathrm{E}_{T} \leftrightarrow h_{1}^{\perp} \\
& \mathrm{E} \leftrightarrow f_{1 T}^{\perp}
\end{aligned}
$$


...but different multi-parton correlations in a suitably defined forward limit (aside from spin for a moment...)



Momentum Space: diagonal in $\times$ non diagonal in $\mathrm{k}_{T}$

$$
\begin{aligned}
E(x, 0, t) & =\int d z^{-} \exp ^{i x P^{+z^{-}}}\left\langle P^{\prime}\right| \bar{\psi}\left(0, z^{-}, 0_{T}\right) \Gamma \psi\left(0,0,0_{T}\right)|P\rangle= \\
& =\sum_{X} \int d z^{-} \exp ^{i\left(x P^{+}-P^{\prime}++P_{x}^{\prime}\right) z^{-}}\left\langle P^{\prime}\right| \bar{\psi}_{+}(0)|X\rangle\langle X| \psi_{+}\left(0,0,0_{T}\right)|P\rangle= \\
& =\int d^{2} k_{T} \phi^{*}\left(x, \underline{k_{T}-\Delta}\right) \phi\left(x, \underline{k_{T}}\right)
\end{aligned}
$$

Coordinate Space: diagonal in $b$

$$
E(x, 0, t)=\int d^{2} b e^{i b \cdot \Delta} \rho(x, b) \quad \mathrm{t}=\Delta^{2}
$$

GPDs and non SSA TMDs are one particle density distributions!


Momentum Space

$$
\begin{aligned}
\left\langle k_{T}^{i}(x)\right\rangle_{U T} & =\int d z^{-} \exp ^{i x P^{+} z^{-}}\langle P| \bar{\psi}\left(0,-z / 2,0_{T}\right) \gamma^{+}\left[\mathcal{W}(-z / 2, z / 2) I_{q}(z / 2)\right] \psi\left(0, z / 2,0_{T}\right)|P\rangle= \\
& =\sum_{X, X^{\prime}} \int d z^{-} \exp ^{i\left(x P^{+}-P^{+}+P_{X}^{\prime}\right) z^{-}}\langle P| \bar{\psi}_{+}(0)|X\rangle\left\langle X^{\prime}\right| \psi_{+}(0)|P\rangle\langle X| \mathcal{W}(0,0) I_{q}(0)\left|X^{\prime}\right\rangle= \\
& =\int d^{2} k_{T} \int d^{2} l_{T} \phi^{*}\left(x, k_{T}\right) \phi\left(x, k_{T}-l_{T}, P\right) \Psi\left(x, l_{T}\right)
\end{aligned}
$$

This has the structure of an unintegrated GPD or GTMD

## Coordinate Space

But careful! Although the correlator factorizes into a GTMD and FSI, it describes multiparton correlations which are different from the TMDs

$$
\left\langle k_{T}^{i}(x)\right\rangle_{U T}=\int d^{2} b_{1} d^{2} b_{1}^{\prime} d^{2} b_{2} \rho_{2}\left[\left(x, b_{1}\right),\left(0, b_{2}\right) ;\left(x, b_{1}^{\prime}\right),\left(0, b_{2}\right)\right] I\left(b_{1}-b_{2}\right)
$$

semi-diagonal (in b) two-particle density distribution

Once we define observables in terms of multiparton density distributions/ correlations what do we do with this information?

Can we gauge what the extent of these multiparticle correlations effect is? (Why is this tour into multiparticle correlations useful?)

Intuitively, we interpret the average $k_{T}$ as measuring:
"the probability of finding a quark correlated with the color field produced by the spectators at a position $z^{-}$away from that quark" M. Burkardt

The multiparticle densities scenario provides a necessary theoretical/formal context/background.

- We understand what makes diquark and quark target models "simple" in this context $\rightarrow$ they are two component models, therefore all of the "complexity" of multiparton interactions is glossed over, FSI is a simple multiplicative factor.
- More realistic diquark type models (with $S$ and $D$ wave spectators, see e.g. Goldstein and Liebl, PL1995; F.Gross and T. Peña, PRD 2011, or taking into account the internal momenta of the spectators, work in progress) could in principle give a very different answer
- Quark models could in principle give a very different answer (see e.g. A. Courtoy and S. Scopetta, PRD 2009)
... on to physically motivated parametrization of data


## $A_{\Lambda^{\prime} \lambda^{\prime}, \lambda \lambda}$ in Diquark Model

$$
\begin{aligned}
& A_{++,++}=\int d^{2} k_{\perp} \phi_{++}^{*}\left(k^{\prime}, P^{\prime}\right) \phi_{++}(k, P) \\
& A_{+-,+-}=\int d^{2} k_{\perp} \phi_{+-}^{*}\left(k^{\prime}, P^{\prime}\right) \phi_{+-}(k, P) \\
& A_{-+,++}=\int d^{2} k_{\perp} \phi_{-+}^{*}\left(k^{\prime}, P^{\prime}\right) \phi_{++}(k, P) \\
& A_{++,-+}=\int d^{2} k_{\perp} \phi_{++}^{*}\left(k^{\prime}, P^{\prime}\right) \phi_{-+}(k, P) . \\
& \phi_{\Lambda, \lambda}(k, P)=\Gamma(k) \frac{\bar{u}(k, \lambda) U(P, \Lambda)}{k^{2}-m^{2}} \\
& \phi_{\Lambda^{\prime} \lambda^{\prime}}^{*}\left(k^{\prime}, P^{\prime}\right)=\Gamma\left(k^{\prime}\right) \frac{\bar{U}\left(P^{\prime}, \Lambda^{\prime}\right) u\left(k^{\prime}, \lambda^{\prime}\right)}{k^{\prime 2}-m^{2}},
\end{aligned}
$$

$$
\begin{gathered}
H=\mathcal{N} \frac{1-\zeta / 2}{1-X} \int d^{2} k_{\perp} \frac{\left[(m+M X)\left(m+M \frac{X-\zeta}{1-\zeta}\right)+\mathbf{k}_{\perp} \cdot \tilde{\mathbf{k}}_{\perp}\right]}{\left(k^{2}-M_{\Lambda}^{2}\right)^{2}\left(k^{\prime 2}-M_{\Lambda}^{2}\right)^{2}}+\frac{\zeta^{2}}{4(1-\zeta)} E, \\
E=\mathcal{N} \frac{1-\zeta / 2}{1-X} \int d^{2} k_{\perp} \frac{-2 M(1-\zeta)\left[(m+M X) \frac{\tilde{k} \cdot \Delta}{\Delta_{\perp}^{2}}-\left(m+M \frac{X-\zeta}{1-\zeta}\right) \frac{k_{\perp} \cdot \Delta}{\Delta_{\perp}^{2}}\right]}{\left(k^{2}-M_{\Lambda}^{2}\right)^{2}\left(k^{\prime 2}-M_{\Lambda}^{2}\right)^{2}} \\
\tilde{H}=\mathcal{N} \frac{1-\zeta / 2}{1-X} \int d^{2} k_{\perp} \frac{\left[(m+M X)\left(m+M \frac{X-\zeta}{1-\zeta}\right)-\mathbf{k}_{\perp} \cdot \tilde{\mathbf{k}}_{\perp}\right]}{\left(k^{2}-M_{\Lambda}^{2}\right)^{2}\left(k^{\prime 2}-M_{\Lambda}^{2}\right)^{2}}+\frac{\zeta^{2}}{4(1-\zeta)} \tilde{E} \\
\tilde{E}=\mathcal{N} \frac{1-\zeta / 2}{1-X} \int d^{2} k_{\perp} \frac{-\frac{\tilde{L}(1-\zeta)}{\zeta}\left[(m+M X) \frac{\tilde{k} \cdot \Delta}{\Delta_{\perp}^{2}}+\left(m+M \frac{X-\zeta}{1-\zeta}\right) \frac{k_{\perp} \cdot \Delta}{\Delta_{\perp}^{2}}\right]}{\left(k^{2}-M_{\Lambda}^{2}\right)^{2}\left(k^{\prime 2}-M_{\Lambda}^{2}\right)^{2}}
\end{gathered}
$$

## Reggeization

$$
\int_{0}^{\infty} d M_{X}^{2} \rho_{R}\left(M_{X}^{2}\right) H(X, 0,0) \sim X^{-\alpha(0)-1}
$$




Brodsky, Close, Gunion $\rightarrow$ DIS ('70s)
Gorshteyn \& Szczepaniak (PRD, 2010)
Brodsky, Llanes, Szczepaniak arXiv:0812.0395

## Crossing Symmetries



## Parametric Form

$$
F(X, \zeta, t)=\mathcal{N} G_{M_{X}, m}^{M_{\Lambda}}(X, \zeta, t) R_{p}^{\alpha, \alpha^{\prime}}(X, \zeta, t)
$$

We asked the question: "What is the minimal number of parameters necessary to fit $X$ and $t$ ?" Can be addressed with Recursive Fit

| Parameters | $H$ | $E$ | $\widetilde{H}$ | $\widetilde{E}$ |
| :---: | :---: | :---: | :---: | :---: |
| $m_{u}(\mathrm{GeV})$ | 0.420 | 0.420 | 2.624 | 2.624 |
| $M_{X}^{u}(\mathrm{GeV})$ | 0.604 | 0.604 | 0.474 | 0.474 |
| $M_{\Lambda}^{u}(\mathrm{GeV})$ | 1.018 | 1.018 | 0.971 | 0.971 |
| $\alpha_{u}$ | 0.210 | 0.210 | 0.219 | 0.219 |
| $\alpha_{u}^{\prime}$ | $2.448 \pm 0.0885$ | $2.811 \pm 0.765$ | $1.543 \pm 0.296$ | $5.130 \pm 0.101$ |
| $p_{u}$ | $0.620 \pm 0.0725$ | $0.863 \pm 0.482$ | $0.346 \pm 0.248$ | $3.507 \pm 0.054$ |
| $\mathcal{N}_{u}$ | 2.043 | 1.803 | 0.0504 | 1.074 |
| $\chi^{2}$ | 0.773 | 0.664 | 0.116 | 1.98 |
| $m_{d}(\mathrm{GeV})$ | 0.275 | 0.275 | 2.603 | 2.603 |
| $M_{X}^{d}(\mathrm{GeV})$ | 0.913 | 0.913 | 0.704 | 0.704 |
| $M_{\Lambda}^{d}(\mathrm{GeV})$ | 0.860 | 0.860 | 0.878 | 0.878 |
| $\alpha_{d}$ | 0.0317 | 0.0317 | 0.0348 | 0.0348 |
| $\alpha_{d}^{\prime}$ | $2.209 \pm 0.156$ | $1.362 \pm 0.585$ | $1.298 \pm 0.245$ | $3.385 \pm 0.145$ |
| $p_{d}$ | $0.658 \pm 0.257$ | $1.115 \pm 1.150$ | $0.974 \pm 0.358$ | $2.326 \pm 0.137$ |
| $\mathcal{N}_{d}$ | 1.570 | -2.800 | -0.0262 | -0.966 |
| $\chi^{2}$ | 0.822 | 0.688 | 0.110 | 1.00 |

## Compton Form factors vs. $\zeta$


$Q^{2}=2 \mathrm{GeV}^{2}$

25
20
15
10
5
0

## Polynomiality!

Goldstein et al. arXiv:1012.3776


## Comparison with lattice





## Implementing DVCS data...




Hall A

Having fitted Jlab data, we predict Hermes
Goldstein et al. arXiv:1012.3776




New flavor separated data on form factors (G.Cates et al., PRL 2011)





After
...Before

## Interpretation?

We believe it is not a simple model (pointlike diquark excluded), but closer to hypothesis of u-quark keeping at larger distances from remnants, than d-quark (Z.E. Meziani)

What determines the interquark distances? Wigner distribution studies Osvaldo Gonzalez Hernandez, S.L.


The partonic configurations radii are determined by an interplay of the Regge and diquark terms





## Pseudoscalar Mesons Electroproduction

$\pi^{\circ}$ and $\eta$ production probing the GPD chiral-odd sector Goldstein et al., arXiv:hep-ph/1201.6088
Issue in a nutshell:
"Collinear factorization approach" for chiral-even process

$$
\begin{aligned}
& g_{0,+; 0,+} \approx \frac{1}{Q} \int d \tau \frac{\phi_{\pi}(\tau)}{\tau} C^{+} \Rightarrow \frac{d \sigma_{L}^{\text {even }}}{d t} \propto \frac{1}{Q^{6}} \\
& g_{1,+; 0,+} \approx \frac{1}{Q^{2}} \int d \tau \frac{\phi_{\pi}(\tau)}{\tau} C^{+} \Rightarrow \frac{d \sigma_{T}^{\text {even }}}{d t} \propto \frac{1}{Q^{8}}
\end{aligned}
$$

"Collinear factorization approach" for chiral-odd process

$$
g_{0+, 0-} \approx \frac{d \sigma_{L}^{\text {odd }}}{d t} \propto \frac{1}{Q^{10}}, \quad g_{1+, 0-} \approx \frac{d \sigma_{T}^{\text {odd }}}{d t} \propto \frac{1}{Q^{8}} .
$$

Transverse component seems to be larger than naively expected

$$
f_{\Lambda_{2}, \Lambda_{i} ; 0, \Lambda^{\prime}}(\xi, t)=\sum_{\lambda_{1} \lambda^{\prime}} \int d x d^{2} k_{\perp} \overline{g_{\Lambda_{\nu}, \lambda_{i} ;, \lambda^{\prime}}}\left(x, k_{\perp}, \xi, t\right) \sqrt{A_{\Lambda^{\prime}, \lambda^{\prime}, \Lambda, \lambda}}\left(x, k_{\perp}, \xi, t\right)
$$



T $\quad g_{T}=g_{\pi}^{\text {odd }}(Q)\left[\frac{1}{x-\xi+i \epsilon}+\frac{1}{x+\xi-i \epsilon}\right]=g_{\pi}^{\text {odd }}(Q) C^{+}$
$g_{L}=g_{\pi}^{o d d}(Q) \sqrt{\frac{t_{o}-t}{Q^{2}}}\left[\frac{1}{x-\xi+i \epsilon}+\frac{1}{x+\xi-i \epsilon}\right]=g_{\pi}^{o d d}(Q) \sqrt{\frac{t_{o}-t}{Q^{2}}} C^{+}$,
$f_{1}=f_{1+, 0+}=g_{1+, 0-} \otimes A_{+-,++}$
$\top \quad \begin{aligned} & f_{2}=f_{1+, 0-}=g_{1+, 0-} \otimes A_{--,++} \\ & f_{3}=f_{1-, 0+}=g_{1+, 0-} \otimes A_{+-,-+}\end{aligned}$
$f_{5}=f_{0+, 0-}=g_{0+, 0-} \otimes A_{--,++}$
$f_{4}=f_{1-, 0-}=g_{1+, 0-} \otimes A_{--,-+}{ }^{35}$

L $f_{6}=f_{0+, 0+}=g_{0+, 0-} \otimes A_{+-,++}$,

$$
\begin{aligned}
& \frac{d^{4} \sigma}{d \Omega d \epsilon_{2} d \phi d t}= \Gamma\left\{\frac{d \sigma_{T}}{d t}+\epsilon_{L} \frac{d \sigma_{L}}{d t}+\epsilon \cos 2 \phi \frac{\overline{d \sigma_{T T}}}{d t}+\sqrt{2 \epsilon_{L}(\epsilon+1)} \cos \phi \frac{d \sigma_{L T}}{d t}\right. \\
&\left.+h \sqrt{2 \epsilon_{L}(\epsilon-1)} \frac{d \sigma_{L^{\prime} T}}{d t} \sin \phi\right\}, \\
& \frac{d \sigma_{T}}{d t}=\mathcal{N}\left(\left|f_{1}\right|^{2}+\left|f_{2}\right|^{2}+\left|f_{3}\right|^{2}+\left|f_{4}\right|^{2}\right) \\
& \frac{d \sigma_{L}}{d t}=\mathcal{N}\left(\left|f_{5}\right|^{2}+\left|f_{6}\right|^{2}\right), \\
& \frac{d \sigma_{T T}}{d t}=2 \mathcal{N} \Re e\left(f_{1}^{*} f_{4}-f_{2}^{*} f_{3}\right) . \\
& \frac{d \sigma_{L T}}{d t}=2 \mathcal{N} \Re e\left[f_{5}^{*}\left(f_{2}+f_{3}\right)+f_{6}^{*}\left(f_{1}-f_{4}\right)\right] \\
& \frac{d \sigma_{L T^{\prime}}}{d t}=2 \mathcal{N} \Im m\left[f_{5}^{*}\left(f_{2}+f_{3}\right)+f_{6}^{*}\left(f_{1}-f_{4}\right)\right]
\end{aligned}
$$

## In terms of GPDs

$$
\begin{aligned}
& \epsilon_{T}^{\mu} T_{\mu}^{\Lambda \Lambda^{\prime}}=e_{q} \int_{-1}^{1} d x \frac{g_{T}}{2 \bar{P}^{+}} \bar{U}\left(P^{\prime}, \Lambda^{\prime}\right)\left[i \sigma^{+i} H_{T}^{q}(x, \xi, t)+\frac{\gamma^{+} \Delta^{i}-\Delta^{+} \gamma^{i}}{2 M} E_{T}^{q}(x, \xi, t)\right. \\
& \left.\frac{\bar{P}^{+} \Delta^{i}-\Delta^{+} \bar{P}^{i}}{M^{2}} \widetilde{H}_{T}^{q}(x, \xi, t)+\frac{\gamma^{+} \bar{P}^{i}-\bar{P}^{+} \gamma^{i}}{2 M} \widetilde{E}_{T}^{q}(x, \xi, t)\right] U(P, \Lambda),
\end{aligned}
$$

## M. Diehl, 2001

$$
\mathcal{H}_{T}, \mathcal{E}_{T}, \tilde{\mathcal{E}}_{T}, \overline{\mathcal{E}}_{T}
$$

$$
\begin{aligned}
& \frac{d \sigma_{T}}{d t} \approx \mathcal{N}^{2}\left[g_{\pi}^{\text {odd }}(Q)\right]^{2} \frac{1}{(1+\xi)^{4}}\left[\left|\mathcal{H}_{T}\right|^{2}+r\left(\left.\overline{\mathcal{E}}_{T}\right|^{2}+\left|\widetilde{\mathcal{E}}_{T}\right|^{2}\right)\right] \\
& \left.\frac{d \sigma_{L}}{d t} \approx \mathcal{N}^{2}\left[g_{\pi}^{\text {odd }}(Q)\right]^{2} \frac{1}{(1+\xi)^{4}} \frac{2 M^{2} \tau}{Q^{2}} \mathcal{H}_{T}\right|^{2} \\
& \frac{d \sigma_{T T}}{d t} \approx \mathcal{N}^{2}\left[g_{\pi}^{\text {odd }}(Q)\right]^{2} \frac{1}{(1+\xi)^{4}}\left[\frac{\tau}{\left[\left.\overline{\mathcal{E}}_{T}\right|^{2}\right.}-\sqrt[\left.\overline{\mathcal{E}}_{T}\right|^{2}]{ }+\frac{\operatorname{Re} \mathcal{H}_{T}}{\operatorname{Re}\left(\overline{\mathcal{E}}_{T}-\mathcal{E}_{T}\right)} \frac{2}{2}+\frac{\left.S m \mathcal{H}_{T}\right]}{2}\right. \\
& \frac{d \sigma_{L T}}{d t} \approx \mathcal{N}^{2}\left[g_{\pi}^{\text {odd }}(Q)\right]^{2} \frac{1}{(1+\xi)^{4}} 2 \sqrt{\left.\frac{2 M^{2} \sqrt{\tau}}{Q^{2}} \mathcal{H}_{T}\right|^{2}} \\
& \frac{d \sigma_{L^{\prime} T}}{d t} \approx \mathcal{N}^{2}\left[g_{\pi}^{\text {odd }}(Q)\right]^{2} \frac{1}{(1+\xi)^{4}} \sqrt{\frac{2 M^{2} \tau}{Q^{2}}}\left[\frac{\Re e \mathcal{H}_{T}}{s m\left(\overline{\mathcal{E}}_{T}-\mathcal{E}_{T}\right)} \frac{2}{2}-\frac{\operatorname{se}\left(\overline{\mathcal{E}}_{T}-\mathcal{E}_{T}\right)}{2}\right]
\end{aligned}
$$

$$
T=\left(t_{0}-t\right) / 2 M^{2}
$$

The question is: how do we normalize the GPDs?
Only Physical constraints on the various chiral-odd GPDs are Forward limit

$$
H_{T}(x, 0,0)=q_{\Uparrow}^{\uparrow}(x)-q_{\Uparrow}^{\downarrow}(x)=h_{1}(x)
$$

Form Factors

$$
\begin{aligned}
& \int H_{T}(x, \xi, t) d x=\delta_{T}(t) \\
& \int \bar{E}_{T}(x, \xi, t) d x=\int\left(2 \tilde{H}_{T}+E_{T}\right) d x=\kappa_{T}(t) \\
& \int \tilde{E}_{T}(x, \xi, t) d x=0
\end{aligned}
$$

No direct interpretation of $E_{T}$

In the diquark model the GPDs are related through Parity

| $S=0$ | $S=1$ |
| :---: | :---: |
| $\phi_{\Lambda^{\prime} \lambda^{\prime}}^{*} \phi_{\Lambda \lambda}$ | $\phi_{\Lambda^{\prime} \lambda^{\prime}}\left(\sum_{\lambda^{\prime \prime}} \epsilon_{\mu \lambda^{\prime \prime}} \epsilon_{\nu}^{\prime \lambda^{\prime \prime}}\right) \phi_{\Lambda \lambda}^{\prime}$ |

In order to explain the working of Parity transformations, we write the LHS and RHS of Fig. 1

|  | RHS | LHS |
| :---: | :---: | :---: |
| $S=0$ | $\phi_{\Lambda^{\prime} \lambda^{\prime}}^{*}$ | $\phi_{\Lambda \lambda}$ |
| $S=1$ | $\phi_{\Lambda^{\prime} \lambda^{\prime} \epsilon_{\mu}^{*}}^{\mu} \lambda^{\lambda^{\prime \prime}}$ | $\epsilon_{\nu}^{\lambda^{\prime \prime}} \phi_{\Lambda \lambda}^{\nu}$ |

$$
\begin{aligned}
& \text { Odd } \\
& \text { Even } \\
& A_{++,--}^{(1)}=-\frac{X+X^{\prime}}{1+X X^{\prime}} A_{++,++}^{(1)} \\
& S=0 \\
& A_{+-,-+}^{(1)}=0 \\
& \text { Odd Even } \\
& A_{++,--}^{(0)}=A_{++,++}^{(0)} \\
& A_{++,+-}^{(0)}=-A_{++,-+}^{(0)} \\
& A_{+-,++}^{(0)}=-A_{-+,++}^{(0)} \text {, } \\
& A_{++,+-}^{(1)}=-\sqrt{\frac{\left\langle\tilde{k}_{\perp}^{2}\right\rangle}{X^{\prime 2}+\left\langle\tilde{k}_{\perp}^{2}\right\rangle / P^{+2}}} A_{++,-+}^{(1)} \\
& A_{+-,++}^{(1)}=-\sqrt{\frac{\left\langle k_{\perp}^{2}\right\rangle}{X^{2}+\left\langle k_{\perp}^{2}\right\rangle / P^{+2}}} A_{-+,++}^{(1)}, \\
& A_{+-,-+}^{(0)}=\frac{t_{0}-t}{4 M} \frac{1}{\sqrt{1-\zeta}} \frac{1}{(1-\zeta / 2)} \frac{\tilde{X}}{m+M X^{\prime}}[E-(\zeta / 2) \tilde{E}] \\
& H_{T}^{u}=\frac{3}{2} H_{T}^{S=0}-\frac{1}{6} H_{T}^{S=1} \\
& H_{T}^{d}=-\frac{1}{3} H_{T}^{S=1}
\end{aligned}
$$

## In terms of GPDs

$S=0$

$$
\begin{align*}
\tilde{H}_{T}^{(0)} & =-\frac{M(1-x)}{m+M x} E^{(0)}  \tag{27a}\\
E_{T}^{(0)} & =2\left(1+\frac{M(1-x)}{m+M x}\right) E^{(0)}  \tag{27b}\\
\widetilde{E}_{T}^{(0)} & =0 \tag{27c}
\end{align*}
$$

$H_{T}^{(0)}=\frac{H^{(0)}+\tilde{H}^{(0)}}{2}-\frac{t_{0}-t}{4 M^{2}} \frac{M(1-x)}{m+M x} E^{(0)}(27 \mathrm{~d})$ $S=1$

$$
\begin{aligned}
\widetilde{H}_{T}^{(1)} & =0 \\
E_{T}^{(1)} & =2 E^{(1)}
\end{aligned}
$$

$$
\tilde{E}_{T}^{(1)}=0
$$

$$
H_{T}^{(1)}=-\frac{2 x}{1+x^{2}} \frac{H^{(1)}+\widetilde{H}^{(1)}}{2}
$$

Odd

## Even







$$
\begin{aligned}
& \sum_{\Lambda} \Im m F_{\Lambda+, \lambda-} \propto h_{1}^{\perp}\left(x, k_{T}^{2}\right) \\
& \sum_{\lambda} \Im m F_{+\lambda,-\lambda} \propto f_{1 T}^{\perp}\left(x, k_{T}^{2}\right) \\
& \hdashline
\end{aligned} \begin{aligned}
& A_{++,+-}-A_{+,++} \propto 2 \tilde{H}_{T}+E_{T} \\
& A_{++,++}-A_{-,++} \propto E
\end{aligned}
$$



How well do the parameters fixed with DVCS data reproduce $\pi^{\circ}$ electroproduction data?


Hall B data, Kubarovsky\& Stoler, PoS ICHEP 2010


## Vary tensor charge as a parameter to see sensitivity of data



FIG. 9 (color online). (color online) Longitudinal/transverse interference term, $d \sigma_{\mathrm{LT}} / d t$, Eq. (15), plotted vs $-t$ at $Q^{2}=$ $2.3 \mathrm{GeV}^{2}, x_{B j}=0.36$, for different values of the $u$ quarks tensor charge, $\delta u$, used as a freely varying parameter in the GPD approach. The $d$ quark component, $\delta d$ was taken as $\delta d=$ -0.62 , i.e. equal to the central value extracted in the global fit of Ref. [44].
$Q^{2}$ dependence $\rightarrow$ obviously not predicted by collinear factorization
Presence of a large transverse component
"Anomalous" Pion Vertex behavior

## Back up

## Explain large T component



$$
\begin{array}{c|cc}
\widetilde{H}(x, \xi, t)-\widetilde{H}(-x, \xi, t) & 2^{--}, 4^{--}, \ldots & (S=1) \text { Polarized antiquarks } \\
\widetilde{E}(x, \xi, t)-\widetilde{E}(-x, \xi, t) & 1^{+-}, 2^{--}, 3^{+-}, 4^{--}, \ldots & (S=0,1)
\end{array}
$$

All these combinations are possible, therefore...


Now that we have allowed for a large $T$ component, explain the $Q^{2}$ dependence....


Take e.g. the modified perturbative approach

$$
g_{\Lambda_{\gamma^{*}}, \lambda ; 0, \lambda^{\prime}}=\int d \tau \int d^{2} b \hat{\mathcal{F}}_{\Lambda_{\gamma^{*}}, \lambda ; 0, \lambda^{\prime}}\left(Q^{2}, \tau, b\right) \alpha_{S}\left(\mu_{R}\right) \exp [-S] \hat{\phi}_{\pi}(\tau, b)
$$

Spin plays a role LIUTAhmad et al.,PHYSICAL REVIEW D 79, 054014 (2009

$$
\begin{aligned}
& \text { V (L=0) } \\
& \text { A (L=1) } \\
& \text { (L=1) } \\
& \mathrm{x}_{1} \Delta+\widetilde{\mathrm{k}}_{\mathrm{T}}^{(1)}\left\{\begin{array}{l}
\mathrm{y}_{1} q^{\prime}+\widetilde{\mathrm{k}}_{\mathrm{T}}^{(1)}
\end{array}\right. \\
& g_{\Lambda_{\gamma^{*}}, \lambda ; 0, \lambda^{\prime}}^{V}=\int d x_{1} d y_{1} \int d^{2} b \hat{\psi}_{V}\left(y_{1}, b\right) \hat{F}_{\Lambda_{\gamma^{*}}, \lambda ; 0, \lambda^{\prime}}\left(Q^{2}, x_{1}, x_{2}, b\right) \alpha_{S}\left(\mu_{R}\right) \exp \left[-S \int \hat{\phi}_{\pi^{o}}\left(x_{1}, b\right)\right. \\
& g_{\Lambda_{r},+, \lambda ;, \lambda^{\prime}}^{A}=\int d x_{1} d y_{1} \int d^{2} b \hat{\psi}_{A}\left(y_{1}, b\right) \hat{F}_{\Lambda_{r},+, \lambda_{0}, \lambda^{\prime}}\left(Q^{2}, x_{1}, x_{2}, b\right) \alpha_{S}\left(\mu_{R}\right) \exp [-S\} \hat{\phi}_{\pi^{o}}\left(x_{1}, b\right) \\
& V=1^{-}, 2^{--}, 3^{--}, \ldots \quad A=1_{58}^{+-}, 3^{+-}, \ldots
\end{aligned}
$$

## Size of qqbar pair

We obtain a mixture of configurations of different "radii" (and different Q2 dependence)
$V \rightarrow \Pi^{0} \rightarrow$ No change of $O A M, \Delta L=0$
$\mathrm{A} \rightarrow \pi^{\circ} \rightarrow$ One unit change of $\mathrm{OAM}, \Delta \mathrm{L}=1$

Axial vector transition involves Bessel $\mathrm{J}_{1}$
$\psi_{A}^{(1)}\left(y_{1}, b\right)=\int d^{2} k_{T} J_{1}\left(y_{1} b\right) \psi\left(y_{1}, k_{T}\right)$, qqbar pair are more separated!

## Summary of $Q^{2}$ dependence

$\checkmark$ Twist 3 DA has a steeper dependence in the longitudinal variable "x" yields larger contribution
$\checkmark$ This can compensate for the fall off in $\mathrm{Q}^{2}$
$\checkmark$ Spin plays a role
(A connection is possible with A. Radyushkin's et al. interpretation of Babar data more channels including rho production need to be explored)

Role of BSA components: H and H -tilde


Goldstein et al. arXiv:1012.3776

## Preliminary Results (PDFs)

(D. Perry, DIS 2010 and MS Thesis 2010, K. Holcomb, Exclusive Processes Workshop, Jlab 2010)
$\mathrm{Q}^{2}=7.5 \mathrm{GeV}^{2}$





